X-ray spectrum dependence of the magnification of cone-beam CT

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Abstract
Modern industrial computed tomography (CT) scanners mainly use flat-panel detectors (FPD) in cone-beam geometry. Here, all length measurements rely on knowledge of the geometrical magnification. It is assumed that this is an invariant and therefore a calibration measurement can be applied to determine the voxel size in use. The magnification is determined by the ratio of the effective distances from the source to the object and from the source to the detector. The scintillator of FPDs is frequently made of structured CsI with a thickness e.g. of about 600 µm. For photon energies varying from 20 keV to 80 keV, its mean penetration depth changes from 80 µm to 600 µm. The magnification could thus change relatively – depending on the voltage, filter and object material – by some 10⁻⁴. This paper shows experimental data for the X-ray-spectrum-dependent effective magnification of a cone-beam CT system and provides guidance on the relevance of the effects for dimensional CT.

Keywords: cone-beam CT, scintillator, magnification, geometrical model, material dependence

1 Theory
The magnification of a cone-beam CT is given by the ratio of the source to detector distance (SDD) and the source to object distance (SOD) (Fig. 1). Usually, the source is considered to be a point and an FPD is considered as a plane whose SDD has to be determined by calibration with a reference standard. This does not take into account the energy-dependent penetration depth (PD) in the FPD. Putting a test object in the CT leads to a shadow image on the detector. The spectrum of this radiation in the shadow image has a different PD compared to the original spectrum or to the shadow spectrum of the reference standard used in advance for “magnification calibration”. Additional pre-absorbers also change the shadow spectrum and the PD. For example, the errors of inner and outer diameters of a hollow cylinder determined from a single radiograph might differ due to different absorption. After 3D reconstruction this is more complex, but the alteration in radiographic magnification suggests a change also in the 3D case.

2 Calculation
The PD of monochromatic X-rays is considered as the mean value of energy deposition depth in certain matter. The photon attenuation coefficient $\mu$ is the coefficient in Beer’s law describing the exponential decay. For many materials, it can be found in [1]. The mean energy deposition depth for a thick absorber is calculated as $1/\mu$. To calculate the PD of a spectrum, the expression (number of photons per energy interval)/$\mu(E)$ has to be integrated over the photon energy $E$. In Fig. 2 (left) the photon mass attenuation coefficient $\mu(E)/\rho$ is plotted for caesium iodide, which is a scintillator material for many common FPDs. This has to be multiplied with the mass density $\rho$. The density of caesium iodide crystals is 4.51 g/cm³. For photon energies varying from 20 keV to 80 keV, $\mu/\rho$ changes from 27 cm²/g to 3.7 cm²/g and so the PD changes from 80 µm to 600 µm. The scintillator of the FPD in use here is about 600 µm thin, so that even for 80 keV photons, about 1/3 of the radiation escapes from the back. Thus, the estimation of the PD by the inverse of $\mu$ is only rough – the realistic value is smaller and listed in Table 1. Fig. 2 (right) shows calculated absorption spectra [2] for two thin objects made of copper and aluminium. It is calculated for a typical X-ray source condition (W reflection target, 150 kV, 0.5 mm Cu filter). The effect of pre-absorbers made of aluminium with varying thickness is also shown. The diagram is normalized to the same energy content of the spectra. Based on the shadow spectra in Fig. 2, the PDs are calculated by integration over $E$ and listed in Table 1. From that the geometrical scale factor correction is calculated referring to the reference value marked in grey.

<table>
<thead>
<tr>
<th>pre-absorber</th>
<th>0 mm Al</th>
<th>20 mm Al</th>
<th>40 mm Al</th>
<th>0 mm Al</th>
<th>20 mm Al</th>
<th>40 mm Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD in µm</td>
<td>242</td>
<td>258</td>
<td>266</td>
<td>223</td>
<td>243</td>
<td>255</td>
</tr>
<tr>
<td>(scale factor – 1) $\cdot 10^6$</td>
<td>18</td>
<td>32</td>
<td>39</td>
<td>0</td>
<td>18</td>
<td>29</td>
</tr>
</tbody>
</table>

Table 1: Simulated PDs and scale factors for a source-to-detector distance of 1132 mm and a 600 µm thin CsI scintillator.
3 Experiments

Using the procedure described in [3], the average grid distance of a grid structure of drilled holes in 50 µm thick copper foil is determined with a relative precision of $< 1 \cdot 10^{-5}$. Fig. 3 (left) shows measurement results for the grid unit distance (g.u.) in detector pixels over the pre-absorber thickness. No copper filter was used, so values listed in Table 1 represent the upper part of the curve that is increasingly more flattened.

A stronger effect is expected for the reference standard shown in Fig. 3 (right) consisting of five pairs of ball bars with two different sphere materials (ZrO$_2$ and ruby). The average distance of the sphere centres for two different voltage/filter combinations is measured in 3D by CT. Due to the imperfection of the CT, the magnification can change by setting different voltages – the ratio of the lengths for each material should not. Currently, a relative change of 284 ppm (=136 ppm + 148 ppm) has been observed for the two materials: if the CT is calibrated with a ruby sphere standard, a ZrO$_2$ sphere standard might be measured wrongly and scaled incorrectly by up to 0.03%.

Conclusion and outlook

It is shown that, depending on the absorbed spectrum of an object, the radiographic magnification in a cone-beam CT varies relatively in a range of up to a few 10$^{-4}$. That means that an object 100 mm in size could have a measurement error in the range of 10 µm – which is relevant for certain applications. This effect appears when using different X-ray voltages/filters and different materials for standards used for voxel size calibration and workpieces to be measured. Thicker objects represent, in different regions and views, a pre-absorber of varying thickness. For a hollow cylinder, for example, the outer contour is filtered weakly in one part of the projections, and strongly in other part – the inner contour is always filtered strongly. This might explain the different scaling for the inner and outer contours. There may still be further considerations for the transfer from 2D to 3D magnification. More experimental results might make it possible to produce correction tables or software.

References

