**X-ray spectrum dependence of the magnification of cone-beam CT**

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**Abstract**
Modern industrial computed tomography (CT) scanners mainly use flat-panel detectors (FPD) in cone-beam geometry. All CT length measurements rely on knowledge of geometrical magnification. To date, it is been assumed that this is invariant, and that a calibration measurement therefore can be applied to determine the voxel size. The magnification is determined by the ratio of the effective distance of the source to the object and that of the source to the detector. The scintillator of FPDs is frequently made of structured CsI with a thickness of about 600 µm. For photon energies varying from 20 keV to 80 keV, the photon’s mean penetration depth varies from 80 µm to 600 µm. The magnification can thus change relatively – depending on the voltage, filter and object material – by some $10^{-4}$. Additional effects of this size arise from the X-ray spectrum’s change over the source’s spot area. This paper presents experimental data for the X-ray-spectrum-dependent effective magnification of a cone-beam CT system and demonstrates the relevance of the effects as one limit for dimensional CT.

**Keywords:** cone-beam CT, scintillator, magnification, geometrical model, material dependence

1 **Introduction**
To date, there have been no reliable uncertainty budgets for dimensional metrology with CT. The apparent reason for this is that not all influence entities are understood. A great deal of time has been spent on investigations of this topic, such as in [1]. The effect of geometrical deviations of the CT scanner in general can be described analytically. Influences on the one hand from the object’s material and alignment, and on the other hand from the choice of the source’s acceleration voltage, filter, target material and inclination, are only described phenomenologically in most cases. To avoid this problem in practice, attempts have been made to guide CT measurements by means of standardization, such as [2], and to prove the measurement capability by means of international intercomparisons [3, 4]. This has proved that measurement results with lengths of about 100 mm typically vary and show errors of about 20 µm, i.e. $2 \cdot 10^{-4}$, relatively speaking. The main focus of both the simulation software and the measurement procedures (including patents) is the treatment and correction of scattered radiation, of fluorescence in the detector material and of partial backscattering into the detector. All these effects are rotationally symmetric to the virtual pencil beams that the cone beam consists of. For this reason, no direct geometric distortion is produced by those effects, apart from the image – described by the point spread function – being blurred. Due to the threshold (or gradient) estimation, local problems for the surface determination algorithms in the CT software that are caused e.g. by an asymmetric scatter background at the edges, by beam hardening or by Feldkamp artefacts, produce further deviations. They mainly affect the bidirectional length measurement. However, even unidirectional length measurements by means of CT feature relative deviations in the above-mentioned size, although the threshold problem is omitted.

Therefore, the goal of this publication is to devote greater attention to the source and to the detector. Some publications have already explored flat-panel detectors (FPDs, also known as digital detector arrays – DDAs) with a full Monte Carlo simulation in detail. In [5], the point spread function of columnar phosphor screens is calculated. This is important, as many FPDs used today have micro-columnar CsI scintillator screens. In [6], a method for the depth-of-interaction determination of single photons is described. In the following section, the authors will describe how the spectrum-averaged depth of interaction from inclined X-rays leads to deviations in scaling, as well as how an energy-dispersive focal spot at reflection targets contributes to – but with a typically minor value. To date, neither effect has been examined for the case of dimensional measurement using cone-beam CT. The ideas presented in the following paper are mainly geometric in nature, including additional approximations from radiation physics, but only on the basis of pencil beams without scatter. Only radiographic magnification is described here, as it is sufficient for the description of the magnification of a CT volume in radial and polar directions.
2 Physical model

The magnification of a cone-beam CT system is given by the ratio $b/a$ of the source-to-detector distance (SDD) $b$ to the source-to-object distance (SOD) $a$ (Fig. 1). It is commonly accepted that an FPD is considered a plane whose SDD has to be determined by means of calibration to a reference standard. This does not take into account the energy-dependent penetration depth (PD) in the FPD that changes the SDD. Putting a test object into a CT system leads to a shadow image on the detector. The spectrum of this radiation in the shadow image has a different PD compared to the original spectrum or to the shadow spectrum of the reference standard used in advance for magnification calibration. Additional pre-absorbers also change the shadow spectrum and the PD: in general, the spectrum is “hardened” (i.e. the mean photon energy is higher). For example, the errors of the inner and outer diameters of a hollow cylinder determined from a single radiograph may differ due to different levels of absorption. After 3D reconstruction from a multitude of projections, the change in magnification is more complex, but the alteration in radiographic magnification is an estimate for the change that takes place in the 3D case.

The SOD is impacted by another effect which can be understood as a focal “colour fringe” effect at the X-ray source: electrons of energy $E_1$ hitting the target (Fig. 2) have a finite PD, as they are charged particles – in contrast to X-ray photon fluence, which only decreases exponentially with increasing depth. We use the continuous slowing down approximation (CSDA), which means that the electrons lose their energy continuously while propagating straight forward until an energy level of zero is reached. This PD of electrons is listed according to the target material and electron energy as the “CSDA range” in [7], where further information can also be found. An additional good approximation for energies of up to 1 MeV is a linear energy loss over the distance of penetration in the material. Thus, after half of the CSDA range, the electron energy is only of half value. This means that the emitted spectrum changes over the position on the target surface. This is complicated by the increasing distance, as generated photons have to cross through the overlying absorbing target material before leaving it. This phenomenon is dependent on the incident electron angle $\alpha$ and vanishes in the limit of flat incidence. The source position is effectively given by the centre of X-ray intensity emission. Because of the position-dependent source spectrum, the effective position depends on the filtering by the X-ray filter, the surrounding material and the object itself. An exact description of the position shift requires detailed information and a complete Monte Carlo calculation, but it can be observed experimentally – in the case of a reflection target (Fig. 1) – by means of a vertical shift $\varepsilon$ of the image correlated with the change of magnification.
3 Estimation of the effects

3.1 Scintillator penetration depth

The photon attenuation coefficient $\mu$ is the coefficient in Beer’s law that describes the exponential decay. It is usual in dosimetry to tabulate the mass attenuation coefficient $\mu/\rho$ that is normalized to the mass density $\rho$ of the absorber. For many relevant materials, $\mu/\rho$ can be found in [8] as a function over the photon energy. Thus, for a monoenergetic photon beam of energy $E$, the photon fluence $\Phi$ over penetration depth $d$ is:

$$\Phi(E, d) = \Phi(E) \cdot e^{-\mu(E) \cdot d}$$  \hspace{1cm} (1)

The following calculation use the approximation that the energy is deposed locally. No transport of fluorescence, scattered radiation or higher-energy charged particles is considered.

The mean PD $D$ of monochromatic X-rays is considered to be the expected (mean) value of energy deposition depth in the relevant material. For an absorber of length $L$ and at energy $E$, it is calculated as:

$$D(E, L) = \frac{\int_0^L x \cdot e^{-\mu(E) \cdot x} \cdot dx}{\int_0^L e^{-\mu(E) \cdot x} \cdot dx} = \frac{1}{\mu(E)} \cdot \frac{1-((\mu(E) \cdot L + 1) \cdot e^{-\mu(E) \cdot L})}{1-e^{-\mu(E) \cdot L}}$$  \hspace{1cm} (2)

In the case of a thick absorber, it is:

$$\lim_{L \to \infty} D(E, L) = \frac{1}{\mu(E)}$$  \hspace{1cm} (3)

In the case of a thin absorber, it is:

$$\lim_{L \to 0} \frac{D(E, L)}{L} = \frac{1}{2}$$  \hspace{1cm} (4)

Equation 3 means that the inverse photon attenuation coefficient is the expected penetration depth for a thick absorber. Equation 4 means that, for a thin absorber, the centre is the estimated penetration depth, as most radiation escapes from the rear. For an incoming broad photon spectrum on a detector of given thickness $L$, $D(E, L)$ has to be averaged over the photon energy, but weighted with the spectral energy contribution to the energy transport $\Phi(E) \cdot E$ and normalized to the total energy flux. Thus, the effective PD is calculated as:

$$D(L) = \frac{\int_0^{E_{\text{max}}} E \cdot \Phi(E) \cdot D(E, L) \cdot dE}{\int_0^{E_{\text{max}}} E \cdot \Phi(E) \cdot dE}$$  \hspace{1cm} (5)

For the following simulation of the effective PD of the shadow images of a thin metal foil, spectra with these parameters were generated with the parameters used later in the experiments:

Target material: tungsten; acceleration voltage: 75 kV/150 kV/225 kV; incident electron beam angle: 20°; target angle: 25°; filter: 0-2 mm of copper and 0-22.5 mm of aluminium (as alternative filters); 0.75 mm of aluminium for the detector cover, 3 mm of plastic for the foil holder and additional cover, 1130 mm of air and 0.8 mm of beryllium for the radiation window.

In practice, the X-ray emission spectrum was generated by means of the spectrum calculator of the aRTist simulation software package version 2.8.1 created by the Bundesanstalt für Materialprüfung und -forschung (BAM) [9, 10, 11] and exported as a spectral fluence table with binning of 1 keV. Further spectra were generated with an additional single absorber of 50 µm of Cu, 50 µm of Al, 1 mm of plastic, 0.8 mm of Be and 1130 mm of air. The quotient between the original spectrum and the attenuated spectrum gives the attenuation caused by the layer. Multiplying the original spectrum by a product of these attenuation tables gives the attenuated spectrum of arbitrary multiples of the layer thickness; attenuation curves over absorber thickness can thus be generated easily. The difference between two such spectra – one with a thin absorber, the other without – gives the shadow spectrum $\Phi(E)$. Finally, the integral of Equation 5 is executed as a sum over the photon energies, assuming $L = 600 \mu$m and assuming $\rho$ as the density of compact caesium iodide crystals (4.51 g/cm$^3$). Figure 3, on the left-hand side, shows the interpolated values of $\mu/\rho$ for caesium iodide. It should be noted that, from 20 keV to 80 keV, $\mu/\rho$ changes from 27 cm$^2$/g to 3.7 cm$^2$/g, thereby changing the effective PD in thick material from 80 µm to 600 µm. Thus, even for 80 keV photons, about 1/3 of the radiation escapes from the rear of the detector. The resulting effective PDs over the filter thickness for all four combinations of aluminium and copper as a filter and an (thin) absorber material are shown in Figure 3 for the three different voltages. It is important to note that the more “hardened” radiation with the copper-foil absorber is more strongly attenuated in the scintillator than the radiation with the aluminium-foil absorber. This is because it contains more photons with energies >33 keV, where $\mu(E)$ of caesium iodide increases due to its K-shell resonances. Hence, the effective penetration depth of aluminium absorbed radiation (without a filter) is higher than for copper absorbed radiation with an increasing voltage (see Fig. 3).
focal spot at the target has to be calculated quantitatively in order to predict the complete effect. At this stage there should only be an estimate of size. From [7], the values of the CSDA range are known and give the PD by dividing it by the density \( \rho \) of 19.25 g/cm\(^3\) in the case of a tungsten target.

![Graph showing \( \mu / \rho \) vs photon energy in keV]

### 3.2 Focal colour fringe effect

As mentioned in Section 2, a complete Monte Carlo simulation of combined electron and photon transport from inside the focal spot at the target has to be calculated quantitatively in order to predict the complete effect. At this stage there should only be an estimate of size. From [7], the values of the CSDA range are known and give the PD by dividing it by the density \( \rho \) of 19.25 g/cm\(^3\) in the case of a tungsten target.

<table>
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<th>150 keV</th>
<th>225 keV</th>
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<tr>
<td>CSDA in g/cm(^2)</td>
<td>0.01872</td>
<td>0.05668</td>
<td>0.1059</td>
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<tr>
<td>PD in ( \mu m )</td>
<td>9.7</td>
<td>29.4</td>
<td>55.0</td>
</tr>
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</table>

Table 1: CSDA range from [7]. Values for 75 keV and 225 keV interpolated.

This represents an increase that is slightly more than linear in relation to the energy. For an electron beam of 150 keV electron energy, it means that the electrons are deaccelerated to 75 keV at a depth of 29.4 \( \mu m \) - 9.7 \( \mu m = 19.7 \mu m \). As the emission power decreases rapidly with the decreasing electron energy, this is an upper estimate for the emitting spot line size (see Fig. 2 from energy \( E_1 \) to \( E_2 \)). For an incident angle of 20° and a target angle of 25°, as is relevant for the experimental study performed here, the apparent height \( e/a \) perpendicular to the direction of the emission is then \( \sin(45°) \cdot 19.7 \mu m = 14 \mu m \) at the target – this could be seen as an upper limit for the effect.
4 Experiments

For the experiments, an industrial metrological computed tomograph of the type NIKON MCT 225 with a tungsten reflection target was used. This tomograph is equipped with a cabin temperature regulator that maintains the temperature inside the centre of the cabin at 20.0 °C and is stable within an interval of ±0.1 °C. The FPD is of the type Perkin Elmer 1620 AN3CS with a detector resolution of 2000 x 2000 pixels over an area of 400 mm x 400 mm. Its scintillator is made of microcolumnar CsI directly grown on a silicon photodiode array and read out by an active transistor array. As all of these layers are directly deposited on a glass substrate, no temperature dependency of the magnification was observed. Taking the manufacturer’s information into account, we assumed a thickness of the scintillator of roughly 600 µm and a front cover thickness of about 0.75 mm of aluminium. The whole system is inside a measurement laboratory room at 20.0 °C ±0.1 °C; this allows external stress on the CT structure to be minimized. The target and the focus coil use a second regulated chiller at 20.0 °C ±0.1 °C for incoming coolant fluid. Before the experiments were performed, the acceleration voltage and beam power were maintained constantly for one hour, while the radiation was blocked by a thick filter. The temperatures of the cabin and of the outer focus coil vessel were monitored and showed that, after this waiting time, the temperature is sufficiently in equilibrium.

4.1 Complete CT with demonstration object

To demonstrate the effect of the material on the magnification calibration of the CT system, a complete CT scan of the object, shown in Figure 4 (left), was performed at two different voltage and filter combinations. The object consists of five pairs of ball bars with two different sphere materials (ZrO$_2$ and ruby). As the atomic number of zirconium (40) is rather high, while that of aluminium (13) is rather low, they absorb different parts of the X-ray spectrum with different PDs in the detector. Surface determination takes place at two different threshold values – but is not overly relevant for the position of the centres. The measured distances of the opposing spheres (Fig. 4, centre) and their average for each material are listed in Figure 4 on the right-hand side.

Due to the limits in the CT system design, the magnification can change slightly by setting different voltages; however, the ratio of the lengths for each material should not. Currently, a relative change of 284 ppm (=136 ppm + 148 ppm) has been observed for the two materials: if the CT is calibrated with a ruby sphere standard, a ZrO$_2$ sphere standard might be measured wrongly and scaled incorrectly by up to 0.03%.

<table>
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<td>1</td>
<td>34.6684</td>
<td>34.6664</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>34.6299</td>
<td>34.6234</td>
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<td>3</td>
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<td>4</td>
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</tr>
<tr>
<td></td>
<td>5</td>
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<td>34.6460</td>
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<td></td>
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</tr>
<tr>
<td>average:</td>
<td></td>
<td>35.0680</td>
<td>35.0732</td>
</tr>
</tbody>
</table>

Figure 4: The figure shows the multiple-ball bar star with 2×5 pairs of carbon-fibre rods with ruby and ZrO$_2$ ceramic spheres together with €5.01 serving as scale. In the centre table, the CT-measured distances of opposite spheres are listed. The relative change of the average of the distances for the both different materials is summarized in the table on the right.

4.2 Systematic investigation of radiographic magnification

In the subsequent subsections, first the test chart used in the experiments is described, followed by the measurement procedure. In the second subsection, the determination of a scale factor with grid charts is briefly introduced, with attention given to the differences to a prior publication of the authors above [11]. The power of the method is shown in the third part by a repeatability measurement performed overnight. The following two subsections investigate the scintillator penetration depth effect and the colour fringe effect by means of a series of measurements with different filter thicknesses. In the experiments, the surrounding material and the filter (Fig. 1) are merged into one filter from the same material for simplicity. Mixed filters have to be additionally simulated/measured in a future work. In this paper, four different combinations of filter/object material are considered from the combination of aluminium and copper.
4.2.1 Grid-like test chart

Two metal foils are drilled using a CNC machine while clamped between two plastic plates. They are arranged in a 40 x 40 equidistant grid with a distance of 1.5 mm and a diameter of 1.0 mm. One foil is made of 50 µm-thick electrolyte copper and the other of 50 µm-thick pure aluminium (>99 %, aluminium alloy 1200). This aluminium alloy contains up to 1 % mass of iron and copper corresponding to about 0.3 % of effective foil thickness. As copper (similarly to iron) absorbs, at the same foil thickness, about ten times more than aluminium, the absorption of the iron/copper foil might have a contribution of up to 3 % of the total absorption, which is tolerable. The metal foils are attached with tape to a 2 mm-thick polycarbonate plate, which is clamped at the rotary stage near its axis (see Fig. 5). Perpendicularity between the magnification axis and the metal foil is adjusted in this way: it is first adjusted to grazing incidence, seen on the detector, and then rotated by 90 degrees. Figure 6 shows a 16-bit digital radiography at 150 kV of both foils without a filter at a magnification of about 4.5. Non-attenuated radiation produces a value of about 55000 digits; the black level is about 4400. The difference image is shown between two subsequent images, one with the object and one with the object removed by using of the transversal stage of the CT. This procedure avoids the common vendor-specific “shading correction” and considers potential drifting of the source better. In the case shown below, the ratio between the pixel noise inside the bores and the difference in the grey value between the bores and the surrounding foil is about 31 in the case of the aluminium foil and 139 in the case of the higher-contrast copper foil. In the following sections, this ratio will be called signal-to-noise ratio, abbreviated to SNR.
4.2.2 Pattern recognition

The method for the determination of the scale factor based on the radiographic measurement of regular 2D grid structures is sufficiently described in [12]. It uses pattern recognition in a digital image. In contrast to the procedure described in [12], here a synthetically produced search pattern is used (Fig. 7, top left). This enhances the pattern recognition if the image is noisy. This pattern can easily be generated, as the hole distance is known to be 1.5 mm and the hole diameter is 1.0 mm, whereas the hole distance in the digital image is about 34.0 pixels, making the diameter $2/3 \times 34$ pixels $= 26.7$ pixels. The recognized positions of the centres of the holes (Fig. 7, bottom left) are fitted to an equidistant grid with free rotation and trapezoidal distortion by means of the Levenberg-Marquardt (LM) optimization. This gives the scale factors of about 34 pixels per grid unit in the horizontal $x$- and in the vertical $y$-direction. The $x$-direction value is called the “scale factor” in the following sections. By turning the rotary stage by 180 degrees, and by producing a second scale factor, the reciprocal of the absolute scale factor at the position of the axis can be calculated as the average of the reciprocals of both the scale factors. For this work, only relative measurements were relevant; for this reason, the second measurement was omitted. In Figure 7, on the right-hand side, the error map is shown as a vector error plot. The radial deviations from an ideal equidistant grid correspond to a maximum of about 0.2 pixels on the detector. This is due to the fact that the foil is wavy, and that variation in source-to-object distance produces variation in the magnification. This static effect can be ignored or corrected: the deviations measured in the first image with a high SNR are subtracted as a correction from the positions of the subsequent images obtained by the pattern recognition. With the corrected grid positions, the LM optimization is more stable.

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![Pattern recognition](image)

Figure 7: Pattern recognition (bottom left) with a synthetic master pattern (top left) and error map (right). The deviations are magnified in such a way that one grid unit of deviation corresponds to one pixel of the detector (magnified 34 times). Deviations of the hole centres remain smaller than 0.2 pixels.
4.2.3 Repeatability
At a voltage of 75 kV without a filter, an integration time of 93 s, and using the copper foil, an SNR of about 152 is reached; at 150 kV, even an SNR of 450 is reached. Figure 8 shows the result of a measurement that took 12 hours to perform. The evaluation of the point-to-point noise gives – as standard deviation – a relative result of only 2.3 \times 10^{-7} in the scale factor. For the determination of the centre position of the grid, the point-to-point noise is 2.6 \times 10^{-4} pixels. Normally, this resolution could only be reached with a laser interferometer, which may also be the only tool capable of investigating the residual drift by independent means. We assume, as described in [12], that the point-to-point noise is generated by the pattern recognition and that it increases reciprocally to the decreasing SNR. This calculated value will be used later on for the error bars, based on a relative value of 2.3 \times 10^{-7} at an SNR of 152. The high resolution makes it possible to use the X-ray source without changing its working conditions while strongly varying the filter attenuation. The lowest SNR used – 1.5 – still gives a small contribution to an uncertainty budget estimated as relative 2.3 \times 10^{-5} due to the scale determination.

![Figure 8: Long-term measurement performed overnight for demonstration of the repeatability and determination of an error estimate.](image_url)

4.2.4 Filter dependence of the magnification
For the measurement of the dependence of the filter on the magnification, the following procedure was applied:
1. Determination of sampling time, so that dynamic range is used optimally. The power for 75 kV, 150 kV and 225 kV is 20 W, 38 W and 40 W, respectively. The integration time of a single frame is 1415 ms, 354 ms and 267 ms. The averages are over 64, 256 and 256 frames, respectively.
2. Warming up of the X-ray source under a full load (see point 1) with a 2 mm copper filter. This is moved, with an auxiliary linear stage in front of the radiation exit, and at the same time the test chart removed from the image by the CT’s x-stage. This is done so that no pattern is burned into the detector.
3. The filter is removed.
4. The test chart is moved into the field of view. Two subsequent images are sampled and each is saved.
5. The test chart is removed from the field of view. A white image is sampled and saved.
6. Another filter is moved into the image and point 4 is accessed until all measurements are done.
7. The filter order is (numbers are mm thickness): no filter, 0.5 Cu, 1 Cu, 1.5 Cu, 2 Cu, 2 Cu, 1.5 Cu, 1 Cu, 0.5 Cu, no filter, 4.5 Al, 9 Al, 13.5 Al, 18 Al, 22.5 Al, 22.5 Al, 18 Al, 13.5 Al, 9 Al, 4.5 Al, no filter.

The uncertainty of scale determination additionally includes the positional repeatability of the x-direction stage and its effects on the magnification and vertical position. By reversing the forward/backward direction in the order of the filters, every measurement is performed a second time. In this way, a dominant effect from the stage or drift of the X-ray source can be excluded. As stated above, the error bars represent the reciprocal SNRs of the evaluated grid structures of the data points shown in Figure 9. The aluminium foil absorbs weakly and only gives a reasonable SNR at a voltage of 150 kV. At 225 kV, it is possible that the scattered radiation from the filter will be a disturbance.
Figure 9: Scale factors in pixel/grid unit dependent on grid material and filter material/thickness. Left: copper grid, right: aluminium grid. 75 kV, 150 kV, 225 kV from top to bottom. The black/red data points indicate dependency from Al/Cu filter thickness, the blue arrows show the relative variation. Subsequent measurement points are connected by lines. Error bars are deduced from Figure 8 and the SNR.
4.2.5 Link between source spot position and magnification

To demonstrate the link between the vertical position of the image and the magnification on the one hand, and to determine the target angle on the other hand, an additional experiment is shown in Figure 10. The vertical dipole magnet of the CT’s microfocus X-ray tube is used to intentionally deflect the position of the electron beam on the target. Here, “y-shift” means a vendor-specific value that is proportional to the respective dipole current. The comparison over the y-shift allows the correlation to be determined between the position shift of the image and the change in magnification by a factor (1+κ). The shift \( \Delta \) in the direction of the magnification axis is calculated from

\[
\frac{b+\Delta}{a+\Delta} = (1 + \kappa) \frac{b}{a}
\]

(6)

\[
\Delta = \frac{ab}{a-b-ab} \cdot \kappa \approx -\frac{ab}{b-a} \cdot \kappa
\]

(7)

For the experiment, \( a \approx 258 \text{ mm} \), \( b \approx 1137 \text{ mm} \), \( \kappa \approx 2.1 \cdot 10^5 \); thus, \( \Delta \approx -7.0 \text{ µm} \). For the same experiment, the vertical shift of the image \( \varepsilon \approx 0.08 \text{ pixel} \cdot 200 \text{ µm/pixel} = 16 \text{ µm} \). With the relation given in Figure 10, on the right-hand side, it follows that the target angle \( \beta \approx 25^\circ \). For a given angle of 135\(^\circ\) for the angle between the electron beam and the magnification axis, it follows that \( \alpha \approx 20^\circ \) for the incident angle. This is plausible and corresponds to the vendor’s information of a (nearly) symmetric constellation.

Figure 10: Vertical deflection of the electron beam at 75 kV with a dipole magnet leads to a change in scale and also to a change in the y-position of the image. The sketch on the right illustrates the relation between the vertical position and the displacement \( \Delta \) in the source position in the direction of the magnification axis in the case of an inclined reflection target.

From the measurement shown in Figure 9 at a voltage of 150 kV, there is also the vertical position shift plotted in Figure 11. Only a small change is observed for the copper foil corresponding to at maximum \( \Delta \approx -2 \mu m / \tan 25^\circ \approx -4 \mu m \), but for the aluminium foil, a change corresponding to \( \Delta \approx -7 \mu m / \tan 25^\circ \approx -15 \mu m \) was observed. This corresponds to a relative increase in magnification of \( 4.5 \cdot 10^3 \) in addition to the simulated effect of the scintillator penetration effect, shown in Figure 3.

Figure 11: Dependency of the vertical position on the filter material and thickness. The measurement is the same as that shown in Figure 10 at 150 kV. Black/red data points indicate dependency from aluminium/copper filter thickness. One pixel on the detector is 200 µm and the magnification is \( \approx 4.5 \). A substantial change in x-position was experimentally not observed.
4.2.6 Comparison of simulation and experiments

For the comparison of the simulation results and the experimentally measured filter-thickness-dependent effect, the scintillator penetration effect (SPE), the modelled colour fringe effect (CFE) (based on the experimentally observed vertical shift) and the measured value are listed in Table 2. The values represent the change in magnification over the complete span of 2 mm thickness of copper, and 22.5 mm of aluminium, and are given as (correction factor -1) · 10⁻⁵. The sign of both effects is positive and shows an increase in magnification with increasing filter thickness. The sum of CFE and SPE should be equal to the value measured.

<table>
<thead>
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<th>Voltage</th>
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<th>225 kV</th>
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<td>Cu</td>
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<tr>
<td>Al</td>
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Table 2: Overview of total filter effect given in 10⁻⁵ deviation of the magnification correction factor from 1.

If it is assumed that the CFE has the same value for 75 kV and 225 kV (not assessed in this study for experimental reasons) as for 150 kV, then the sum of the effects has a value which correctly describes the measured value with the right sign and also better than a factor of two. The measured course in Figure 9 has a form similar to that simulated in Figure 3 for the SPE. This is still a surprisingly good agreement to the calculation if it is considered how many assumptions about material and geometry influence the simulation and how many simplifications are made. For example, if the scintillator thickness is assumed to be 1 mm instead of 0.6 mm, the simulation will give a slightly higher correction than the one measured. In addition, the X-ray backscatter from the glass substrate of the detector will increase the effective detector penetration depth remarkably. More calculations and experiments will be necessary in further work. For the given magnification of 4.5 used in our study, with the given detector geometry and type, the SPE seems to dominate, but the CFE increases in importance as the source-to-object distance decreases, i.e. at higher magnification.

5 Conclusion and outlook

In this work, systematic experiments on the geometric magnification of CT systems featuring a cone-beam setup and a flat panel detector (FPD) are performed. In the experiments, a high precision image analysis is carried out on radiographic projections of dedicated grid structures [12]. Furthermore, the sensitivity of down to 10⁻⁷ in scale achieved in these experiments is otherwise only known from interferometric measurements. It is shown that, depending on the absorbed spectrum of an object, the radiographic magnification varies relatively in an industrial cone-beam CT in a range of up to a few 10⁻². This means that an object 100 mm in size could have a measurement error of around 10 µm – which is relevant for many applications and an important contribution to the length measurement error budget relevant for a manufacturer to create a CT system specification for dimensional measurements. This is an effect which is not treated as standard when setting up and correcting CT systems. The reason for the observed effect verified here is the physics of the X-ray source and the flat-panel detector: neither represents a point or an area, but rather a volume wherein the radiation is either produced or detected. Depending on the source conditions, on the self-absorption of the object and on the material of the feature to be measured, a different effective source point and effective detection plane are to be taken as a calculation basis. This means that the geometrical magnification – being of utmost importance for any dimensional measurement – is dependent on all of these factors and different for each projection image. This also includes magnification changes that are visible in each projection image, as the individual X-ray beams that hit the individual detector pixel pass through different amounts of material and are thus absorbed and differently modified with respect to their X-ray spectrum. Therefore, this effect appears in the dimensional CT user’s daily practice when using different X-ray voltages and/or X-ray filters and different materials for standards used for voxel size calibration and workpieces to be measured. For a hollow cylinder, for example, the outer contour is filtered weakly in one part of the projections and strongly in other parts – the inner contour is always filtered strongly. This might explain the different scaling for the inner and outer contours, as observed in the past. More experimental results and analytic models might make it possible to produce correction tables or correcting software. This will be the subject of future investigations.

The effects described in this work are of a generic nature. The lateral change of the virtual emission centre that is dependent on the absorption of the emitted X-rays occurs for X-ray systems only using a reflection target. However, for all target types, the effective depth of the emission inside the target has an impact on the geometrical magnification. This effect becomes more relevant for high magnification scenarios. The change of the penetration depth inside a flat-panel scintillator caused by a change of the incident spectrum (as a result of filtering and object absorption) causes a change in the geometrical magnification as well. This change of penetration inside the detector is relevant for cone-beam or fan-beam CT systems that use a planar or
linear detector. Only in the event that a curved detector (realized only for line detectors) with a radius corresponding to the source-detector distance is used, will the effect be cancelled out.

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